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Simulation Study of Modified Two-Parameter Liu Estimator (MTPLE) Method to Overcome Multicollinearity in The Poisson Regression Model

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Abstract

This study aims to evaluate the performance of the Modified Two-Parameter Liu Estimator method in dealing with multicollinearity and compare the performance of Maximum Likelihood Estimator, Liu Estimator, and Modified Two-Parameter Liu Estimator. Simulated data was used with n = 30, 50, 75, 150, and 300 in a Poisson regression model (p = 4, 6, 8) with ρ = 0.89, 0.95, and 0.99. The performance is evaluated using the mean square error criterion. The study results showed the superiority of Modified Two-Parameter Liu Estimator over the other estimators as it has the smallest mean square error value.

Keywords: *Modified Two-Parameter Liu Estimator, Liu Estimator, Maximum Likelihood Estimator, Multicollinearity, Poisson Regression Model.*

I. INTRODUCTION

In regression model analysis, multicollinearity is a phenomenon in which the independent variables show high interdependence. Multicollinearity can cause parameter estimates instability, leading to incorrect conclusions [1]. The Poisson Regression Model is used for analysis of count data [2]. The commonly used method to estimate regression model parameter is Maximum Likelihood Estimator. However, Maximum Likelihood Estimator provides unstable parameter estimates when multicollinearity is present. Some other methods exist for estimating parameters in the regression model with the existence of multicollinearity such as Liu Estimator and Modified Two-Parameter Liu Estimator.

The Liu Estimator method uses parameter d to adjust the variance of the estimator. The study of this method has been done by several researchers. Al-Juboori et al [3] and Saputri et al [4] conducted research on the Liu Estimator method, finding that Liu Estimator is better than Maximum Likelihood Estimator, Ridge Estimator, and LASSO methods in the Poisson Regression Model and Multinomial Logistic Regression. In addition, Modified Two-Parameter Liu Estimator method which uses parameter k which to control the level of bias applied to the estimator [2] has been evaluated by Abonazel et al [5] and developed the Liu Estimator method into Modified Two-Parameter Liu Estimator for handling multicollinearity in the Conway Maxwell – Poisson Regression Model. The study result showed that the Modified Two-Parameter Liu Estimator has a smaller mean square error value than Liu Estimator, which means that Modified Two-Parameter Liu Estimator is better than Liu Estimator.

This study will compare the performance of Maximum Likelihood Estimator, Liu Estimator, and Modified Two-Parameter Liu Estimator to overcome multicollinearity in the Poisson regression model based on the smallest mean square error by a simulation study with different levels of correlation value, some sample sizes, different sum of independent variables, so that the study results will show the performance comparison between Maximum Likelihood Estimator, Liu Estimator, and Modified Two-Parameter Liu Estimator in some various data condition.

II. LITERATURE REVIEW

The commonly used method to estimate the parameter of Poisson Regression Model (PRM) is Maximum Likelihood Estimator (MLE). According to Abdelwahab, et al. [2], the estimation of the $\hat{\beta}$ parameter with MLE can be written mathematically as follows:

$$\hat{\beta}_{PMLE} = (H)^{-1}X^T\widehat{W}\hat{s}$$

Where $H = X^T\widehat{W}X$, $\hat{s}$ is an n-dimensional vector with the ith element $\hat{s}_i = \log\hat{\mu}_i + \frac{(y_i-\hat{\mu}_i)}{\hat{\mu}_i}$, and $\widehat{W} = diag[\hat{\mu}_i]$.

- *Liu Estimator (LE)*

LE is one of the methods that is known to be quite good in dealing with multicollinearity in the PRM. Liu Estimator PLE can be written mathematically as follows:

$$\boldsymbol{\hat{\beta}_{LE} = (H+I)^{-1}(H+dI)\hat{\beta}_{MLE}}$$

Where $\boldsymbol{d}$ is a shrinkage parameter $\mathbf{0 < d < 1}$. According to Qasim, et al. [6], the LE performance depends on the shrinkage parameter ($\boldsymbol{d}$). $\boldsymbol{d}$ is used to shrink the regression coefficients by adjusting the estimator's variance. The optimal value of $\boldsymbol{d}$ can be determined using several approaches as follows:

$$\boldsymbol{D_1 = \max\left(0, \frac{\hat{\alpha}^2_{max} - 1}{\frac{1}{\hat{\lambda}_{max}} + \hat{\alpha}^2_{max}}\right)}$$

$$\boldsymbol{D_2 = \max\left(0, median\left(\frac{\hat{\alpha}^2_i - 1}{\frac{1}{\hat{\lambda}_i} + \hat{\alpha}^2_i}\right)\right)}$$

$$\boldsymbol{D_3 = \max\left(0, \sum_{i=1}^{p}\left(\frac{\hat{\alpha}^2_i - 1}{\frac{1}{\hat{\lambda}_i} + \hat{\alpha}^2_i}\right)/p\right)}$$

$$\boldsymbol{D_4 = \max\left(0, \max\left(\frac{\hat{\alpha}^2_i - 1}{\frac{1}{\hat{\lambda}_i} + \hat{\alpha}^2_i}\right)\right)}$$

$$\boldsymbol{D_5 = \max\left(0, \min\left(\frac{\hat{\alpha}^2_i - 1}{\frac{1}{\hat{\lambda}_i} + \hat{\alpha}^2_i}\right)\right)}$$

Where $\boldsymbol{\alpha = \gamma^t\beta}$, $\boldsymbol{\gamma}$ is an orthogonal matrix with the eigenvector of $\mathbf{X^T\widehat{W}X}$ as the columns, $\boldsymbol{\hat{\alpha}^2_{max}}$ is the maximum element from $\boldsymbol{\alpha_i^2}$, $\boldsymbol{\lambda} = \boldsymbol{X^T\widehat{W}X}$ and $\boldsymbol{\hat{\lambda}_{\mathbf{max}}}$ is the maximum element from $\boldsymbol{X^T\widehat{W}X}$.

- *Modified Two-Parameter Liu Estimator (MTPLE)*

According to Abdelwahab, et al [2], MTPLE is a modification of LE with a pair of parameters (k, d) to provide more stable and accurate estimates even in the presence of multicollinearity. Where the k parameter is used to control the level of bias applied to the estimator, while the d parameter is used to shrink the regression coefficients by adjusting the variance of the estimator. MTPLE can be written mathematically as follows:

$$\hat{\beta}_{\mathrm{MTPLE}} = (\mathrm{H}+\mathrm{I})^{-1}(\mathrm{H} - (\mathrm{k}+\mathrm{d})\mathrm{I})\hat{\beta}_{\mathrm{MLE}}$$

Where the shrinkage parameter $\mathrm{k} > 0$ and $0 < \mathrm{d} < 1$. The optimal value of k can be determined using several approaches as follows:

$$k_{1,d} = k_1^*$$

$$k_{2,d} = k_2^*$$

$$k_{3,d} = \left(\frac{\min(\hat{\lambda}_i)\left(1 - \min\hat{\alpha}_i^2\right)}{1 + \min(\hat{\lambda}_i)\,mean(\hat{\alpha}_i^2)}\right) - D_5$$

$$k_{4,d} = \left(\frac{p+1}{\sum_{i=1}^{p+1}\left[\left(\frac{1}{\hat{\lambda}_i}\right) + 2\hat{\alpha}_i^2\right]}\right) - D_5$$

$$k_{5,d} = \left(\frac{1}{\min(1 + \hat{\lambda}_i\hat{\alpha}_i^2)}\right) - D_5$$

Where k^* is the value of shrinkage parameter used in the Ridge Estimator (RE).

III. METHODS AND MATERIAL

Simulation data with 4, 6, and 8 independent variables (p = 4, 6, 8) with a correlation level between the independent variables of 0.89, 0.95, and 0.99 (ρ = 0.89, 0.95, 0.99) and sample size use is n = 30, 50, 75, 150, and 300 using the Python Programming with 1000 iterations was used in this research. The independent variables are generated from Monte Carlo simulations:

$$X_p = \sqrt{1-\rho^2}\,Q_{ij} + \rho Q_{i,(p+1)}$$

where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, p$, and Q_{ij} are derived from a standard normal distribution, and ρ represents the correlation between independent variables.

The dependent variable is derived from the Poisson distribution with μ_i parameter:

$$y_i \sim Poisson(\mu_i)$$

where μ_i parameter is determined by the exponential function of the x_{ij} variables combination:

$$\mu_i = \exp(\beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip}).$$

The slope coefficients were chosen such that $\beta_0 = 0$ and $\beta_1 = \beta_2 = \cdots = \beta_p = 1$. The variance inflation factor (VIF) is checked to determine whether the data contains multicollinearity. If the VIF value is greater than 10, there is a multicollinearity issue.

The best parameter estimates is analyzed by the MSE value.

$$MSE = \frac{1}{L}\sum_{n=1}^{L}(\hat{\beta}_l - \beta)^T(\hat{\beta}_l - \beta)$$

with L is sum of the simulation iteration. The smaller MSE value means that the method estimates parameter β better than the others.

IV. RESULT AND DISCUSSION

In order to see the performance of Modified Two-Parameter Liu Estimator in dealing with multicollinearity in the Poisson Regression Model, the first step is to calculate the VIF value to see if there is multicollinearity issue between the independent variables. If the VIF value > 10, it shows that there is correlation between the independent variables and it can be believed that there is multicollinearity issue. The VIF value for p = 4 with different n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99 is shown in Table 1.

Table 1 VIF Value for p = 4

ρ	n	VIF			
		X_1	X_2	X_3	X_4
0.89	30	12.010060	10.569560	9.536615	13.421533
	50	7.720681	10.253003	6.110895	5.645170
	75	6.651032	8.328210	8.166649	10.004703
	150	6.381281	6.319610	10.010757	7.335995
	300	10.157246	6.955293	7.049019	6.930528
0.95	30	16.197505	17.978610	16.396906	19.422230
	50	24.820972	17.562423	17.446578	26.842710
	75	13.065858	10.865432	12.898665	12.179608
	150	14.202300	13.926726	19.998441	13.983002
	300	10.672461	14.418109	14.383375	13.181500
0.99	30	81.002232	83.837782	148.022946	85.148178
	50	111.150840	70.549179	124.938623	73.285218
	75	64.310217	59.108888	54.018548	75.560476
	150	82.106168	81.625606	84.574890	90.261308
	300	70.800892	77.127642	77.812566	66.882894

Table 1 shows that the VIF value for the simulation data with p = 4, n = 30, 50, 75, 150, 300 and ρ = 0.89 contain partial correlation between the independent variables because only some independent variables have VIF value > 10. In addition, the VIF value for simulation data with p = 4, n = 30, 50, 75, 150, 300 and ρ = 0.95, 0.99 shows that there is a full correlation since the VIF value > 10 for each independent variable. The same process for p = 6 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99. The result of the VIF value for the analysis can be seen in Table 2.

Table 2 VIF Value for p = 6

ρ	n	VIF					
		X_1	X_2	X_3	X_4	X_5	X_6
0.89	30	10.695424	8.678992	10.016051	13.574706	10.649569	8.500227
	50	10.117790	9.187104	9.977904	8.087119	9.127756	8.084564
	75	7.145195	9.191120	7.893571	10.525175	7.944035	8.572533
	150	8.521462	10.790674	9.018163	7.890169	9.058083	7.469431
	300	10.216786	8.712078	7.322758	8.715245	7.356700	7.873428
0.95	30	36.908850	21.687356	26.489027	17.993873	23.858898	26.539212
	50	13.428877	15.268736	21.139677	17.355469	17.704880	14.993817
	75	16.639441	16.807380	14.681913	16.505771	19.886014	13.710348
	150	16.848742	20.361131	18.958940	22.671450	17.301298	17.880302
	300	14.757964	18.789340	15.280656	16.286249	15.877039	16.779795
0.99	30	57.857252	114.373038	83.947888	89.889682	142.817761	102.173241
	50	74.920310	57.455764	40.237793	51.669702	62.842833	37.664781
	75	74.507093	78.072829	64.440611	55.021186	68.082535	64.331298
	150	73.118838	94.445738	87.422364	80.242013	84.323408	91.381504
	300	93.760924	84.732367	84.718736	94.076082	85.499944	86.457550

Table 2 shows that the VIF value for the simulation data with p = 6, n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99. It shows that the data with ρ = 0.89 contain partial correlation between the independent variables because only some independent variables have VIF value > 10. In addition, the VIF value for simulation data with ρ = 0.95, 0.99 shows that there is a full correlation since the VIF value > 10 for each independent variable. The VIF value for p = 8 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99 is displayed in Table 3.

Table 3 VIF Value for p = 8

ρ	n	VIF							
		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
0.89	30	21.684533	15.904838	20.264510	22.425131	19.556173	26.617014	15.605141	10.881542
	50	9.750551	9.812801	8.079585	10.431655	13.195812	8.162553	6.462205	7.955333
	75	11.389744	9.246713	12.209354	8.389446	9.701891	10.677032	9.218140	8.538410
	150	6.641215	7.747876	9.186800	8.520998	9.464797	10.447705	8.364180	9.194026
	300	8.858627	7.692054	8.270312	8.346181	10.084002	8.379562	7.936028	7.729297
0.95	30	42.400094	30.137946	34.453507	48.320691	62.139111	33.313852	33.053333	40.569969
	50	20.216844	24.739795	18.650281	22.920181	17.099716	19.936648	13.094733	26.490144
	75	23.990971	25.221091	18.020509	24.422031	21.563622	18.950947	20.498645	18.644758
	150	15.726703	15.980549	14.711554	16.477617	14.346789	19.025094	16.403943	14.344638
	300	17.472537	18.099225	16.614676	15.537909	15.196512	17.805207	17.338259	17.096438
0.99	30	90.076811	108.531998	67.104598	245.495906	140.064933	82.273329	93.040405	189.004466
	50	89.601121	136.001334	105.634517	92.217112	129.743801	136.011482	81.622314	109.741842
	75	103.388595	98.419008	99.455527	97.242039	152.180771	114.078873	100.542453	93.807302
	150	94.354811	88.119036	85.320177	96.138264	94.978487	83.044321	98.638202	107.182921
	300	83.999753	82.025563	93.533029	89.373806	81.027324	81.476822	100.565323	84.122885

Table 3 shows that the VIF value for the simulation data with p = 8, n = 30, 50, 75, 150, 300 and ρ = 0.89 also contain partial correlation between the independent variables because only some independent variables have VIF value > 10. In addition, the VIF value for simulation data with p = 8, and n = 30, 50, 75, 150, 300 and ρ = 0.95, 0.99 shows that there is a full correlation since the VIF value > 10 for each independent variable.

According to Table 1-3, it can be concluded that the simulated data with p = 4, 6, 8 and ρ = 0.89 contain partial correlation between the independent variables for n = 30, 50, 75, 150, 300 because only some independent variables have VIF value > 10, and the simulated data with p = 4, 6, 8, n = 30, 50, 75, 150, 300, and ρ = 0.95, 0.99 contain a full correlation since the VIF value > 10 for each independent variable. Therefore, it can be believed that there is multicollinearity issue in this simulation data.

The next step is to calculate the MSE value for the Maximum Likelihood Estimator, Liu Estimator, and Modified Two-Parameter Liu Estimator methods to analyze which is the best parameter β estimate with the muticollinearity issue present in the model. The smaller MSE value means that the method estimates parameter β better than the others. To organize the presentation of the study results, Table 4-6 shows the MSE value for each estimator in the case of p = 4, 6, 8 and the smallest MSE value in each row is highlighted in bold. Table 4 shows the MSE values for p = 4, n = 30, 50, 75, 150, 300, and ρ = 0.89, 0.95, 0.99.

Table 4 MSE Values of MLE, LE, MTPLE Methods for p = 4

ρ	**n**	**MLE**	**LE**			**MTPLE**		
			$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{d_3}$	$\boldsymbol{k, d_1}$	$\boldsymbol{k, d_2}$	$\boldsymbol{k, d_3}$
0.89	30	0.142046	0.096606	0.113438	0.135652	**0.089429**	0.089575	0.090703
	50	0.051997	0.046862	0.048831	0.051301	**0.044842**	0.045715	0.046088
	75	0.014066	0.013426	0.013696	0.013989	**0.013277**	0.013345	0.013836
	150	0.013786	0.012475	0.012605	0.012748	**0.011208**	0.012400	0.012505
	300	0.005443	0.005332	0.005358	0.005366	**0.003311**	0.003347	0.004355
0.95	30	0.242463	0.159602	0.190030	0.230697	**0.145666**	0.162588	0.199749
	50	0.221343	0.148859	0.174548	0.210678	**0.138295**	0.149464	0.171075
	75	0.155748	0.124534	0.136303	0.151438	**0.119137**	0.122867	0.129962

	150	0.074210	0.065974	0.069133	0.073088	**0.064656**	0.066750	0.067640
	300	0.030652	0.028707	0.029526	0.030418	**0.028312**	0.029506	0.029972
0.99	30	0.867345	0.809038	0.789412	0.789339	**0.667541**	0.672944	0.688100
	50	0.732307	0.447078	0.475282	0.578745	**0.399348**	0.405903	0.409717
	75	0.514622	0.421520	0.371671	0.382231	**0.222354**	0.314581	0.337217
	150	0.495940	0.346998	0.360336	0.364248	**0.196868**	0.283732	0.291170
	300	0.322757	0.233413	0.285444	0.314203	**0.172842**	0.219090	0.282066

According to Table 4, Modified Two-Parameter Liu Estimator has smaller MSE value for p = 4 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99 compared to the MSE values for Maximum Likelihood Estimator and Liu Estimator. If we look in detail the Modified Two-Parameter Liu Estimator with parameter k, d_1 has the smallest MSE value. The same process for p = 6 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99. The result of the MSE value can be seen in Table 5.

Table 5 MSE Values of MLE, LE, MTPLE Methods for p = 6

ρ	**n**	**MLE**	**LE**			**MTPLE**		
			$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{d_3}$	$\boldsymbol{k, d_1}$	$\boldsymbol{k, d_2}$	$\boldsymbol{k, d_3}$
0.89	30	0.008395	0.008132	0.008234	0.008360	**0.008077**	0.008112	0.008169
	50	0.007065	0.005286	0.006044	0.006619	**0.003927**	0.004398	0.004887
	75	0.004686	0.002654	0.003267	0.003682	**0.000947**	0.001658	0.002371
	150	0.000585	0.000583	0.000584	0.000585	**0.000582**	0.000583	0.000584
	300	0.000305	0.000302	0.000303	0.000304	**0.000296**	0.000299	0.000301
0.95	30	0.429330	0.144008	0.237789	0.384416	**0.116423**	0.155535	0.184494
	50	0.058231	0.050516	0.053491	0.057232	**0.043314**	0.045639	0.048372
	75	0.045828	0.040198	0.042592	0.045159	**0.034792**	0.036678	0.038738
	150	0.020717	0.019804	0.020180	0.020600	**0.019016**	0.019309	0.019647
	300	0.004460	0.004439	0.004447	0.004457	**0.004434**	0.004438	0.004442
0.99	30	0.788743	0.640089	0.715794	0.776525	**0.399414**	0.460821	0.537255
	50	0.766427	0.410252	0.558603	0.601539	**0.189933**	0.253855	0.302322
	75	0.657797	0.339541	0.472404	0.541224	**0.171917**	0.230722	0.225484
	150	0.545862	0.304777	0.358840	0.391173	**0.168880**	0.221648	0.222687
	300	0.394734	0.243946	0.267621	0.298199	**0.113822**	0.157195	0.187471

Table 5 shows that the MSE value for Modified Two-Parameter Liu Estimator is smaller than MSE value for Maximum Likelihood Estimator and Liu Estimator in the simulation data with p = 6, n = 30, 50, 75, 150, 300, and ρ = 0.89, 0.95, 0.99. If we look in detail, same as the result for the simulation data with p = 4, the Modified Two-Parameter Liu Estimator with parameter k, d_1 has the smallest MSE value compared to the MSE value for Maximum Likelihood Estimator and Liu Estimator. The same process for p = 8 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99. The result of the MSE value is displayed in Table 6.

Table 6 MSE Values of MLE, LE, MTPLE Methods for p = 8

ρ	**n**	**MLE**	**LE**			**MTPLE**		
			$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{d_3}$	$\boldsymbol{k, d_1}$	$\boldsymbol{k, d_2}$	$\boldsymbol{k, d_3}$
0.89	30	0.001851	0.001608	0.001727	0.001846	**0.001152**	0.001358	0.001564
	50	0.001133	0.000572	0.000634	0.000841	**0.000404**	0.000495	0.000676
	75	0.000357	0.000356	0.000356	0.000357	**0.000353**	0.000354	0.000355
	150	0.000289	0.000287	0.000288	0.000288	**0.000284**	0.000285	0.000286
	300	0.000069	0.000034	0.000045	0.000067	**0.000019**	0.000024	0.000046
0.95	30	0.464620	0.320492	0.345832	0.392897	**0.307779**	0.331749	0.387443
	50	0.041277	0.028818	0.036925	0.040365	**0.027826**	0.029289	0.031086
	75	0.014460	0.013736	0.014041	0.014373	**0.013032**	0.013274	0.013542
	150	0.005548	0.005427	0.005480	0.005534	**0.005296**	0.005344	0.005392
	300	0.000782	0.000780	0.000781	0.000782	**0.000778**	0.000779	0.000780
0.99	30	0.786643	0.468302	0.591712	0.622931	**0.225847**	0.315032	0.362025
	50	0.716023	0.428895	0.499863	0.589462	**0.194548**	0.287232	0.328548
	75	0.655724	0.412854	0.465583	0.575986	**0.170307**	0.271460	0.280286
	150	0.629773	0.323738	0.433058	0.545091	**0.165484**	0.220928	0.229085
	300	0.540487	0.318382	0.345730	0.451707	**0.132769**	0.142129	0.194119

Table 6 shows that Modified Two-Parameter Liu Estimator has smaller MSE value for p = 8 with n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99 compared to the MSE values for Maximum Likelihood

Estimator and Liu Estimator. The smallest MSE is also produced by Modified Two-Parameter Liu Estimator with the parameter k, d_1.

According to Table 4-6, Modified Two-Parameter Liu Estimator constantly has smaller MSE value for each simulation data with p = 4, 6, 8, n = 30, 50, 75, 150, 300 and ρ = 0.89, 0.95, 0.99 compared to the MSE values for Maximum Likelihood Estimator and Liu Estimator.

V. CONCLUSION

Based on the study results, it can be concluded that Modified Two-Parameter Liu Estimator gives better parameter β estimate with different levels of correlation (ρ = 0.89, 0.95, 0.99) for all sample sizes studied (n = 30, 50, 75, 150, 300), since the MSE value is the smallest compared to the MSE value for Maximum Likelihood Estimator and Liu Estimator. This means that Modified Two-Parameter Liu Estimator performs better than Maximum Likelihood Estimator and Liu Estimator when the Poisson Regression Model contains multicollinearity issue.

REFERENCES

[1]. Kyriazos, T. & Poga, M. 2023. Dealing with Multicollinearity in Factor Analysis: The Problem, Detections, and Solutions. Journal of Statistics. 13(3): 404-424. https://doi.org/10.4236/ojs.2023.133020

[2]. Abdelwahab, M., Abonazel, M., Hammad, A., & Masry, A. 2024. Modified Two-Parameter Liu Estimator for Addressing Multicollinearity in the Poisson Regression Model. Axioms. 13(46): 1-22. https://doi.org/10.3390/axioms13010046

[3]. Al-Juboori, F. 2022. A new method of Poisson regression estimator in the presence of a Multicollinearity problem: Simulation and application. Journal of Engineering and Natural Sciences. 10(2): 164-188. https://www.elibrary.ru/item.asp?id=48433144

[4]. Saputri, S. A., Herawati, N., Sutrisno, A., Nusyirwan, Nisa, K. 2024. Comparison Of MLE, LASSO, And Liu Estimator Methods to Overcome Multicollinearity in Multinomial Logistic Regression: Simulation Study. Global Scientific and Academic Research Journal of Multidisciplinary Studies. 3(5): 8-12. https://gsarpublishers.com/wp-content/uploads/2024/05/GSARJMS662024-Gelary-script.pdf

[5]. Abonazel, M., Awwad, F., Eldin, E., Kibria, G., & Khattab, I. 2023. Developing A Two-Parameter Liu Estimator for The COM-Poisson Regression Model: Application and Simulation. Journal of Applied Mathematics and Statistics. 9: 1-16. https://doi.org/10.3389/fams.2023.956963

[6]. Qasim. M., Kibria, B. M. G., Mansson, K., & Sjolander, P. 2020. A New Poisson Liu Regression Estimator: method and application. Journal of Applied Statistics. 47(12): 2258-2271. https://doi.org/10.1080/02664763.2019.1707485

[7]. Alrweili, H. 2024. Kibria–Lukman Hybrid Estimator for Handling Multicollinearity in Poisson Regression Model: Method and Application. International Journal of Mathematics and Mathematical Sciences. 2024(1): 1-31. https://doi.org/10.1155/2024/1053397

[8]. Amin, M., Akram, M. N., & Kibria, B. M. G. 2021. A New Adjusted Liu Estimator for The Poisson Regression Model. Wiley. 33(20): 1-12. https://doi.org/10.1002/cpe.6340

[9]. Chan, J., Leow, S., Bea, K., Cheng, W., Phoong, S., Hong, Z., & Chen, Y. 2022. Mitigating the Multicollinearity Problem and Its Machine Learning Approach: A Review. Journal of Mathematics. 10(8): 1-17. https://doi.org/10.3390/math10081283